Some Important Cases

1. **Multiple Optimal Solution;**
   Example: Solve the following LPP by graphical method

$$Max(Z) = 100x_1 + 40x_2$$

Subject to

$$5x_1 + 2x_2 \leq 1000$$
$$3x_1 + 2x_2 \leq 900$$
$$x_1 + 2x_2 \leq 500$$

and

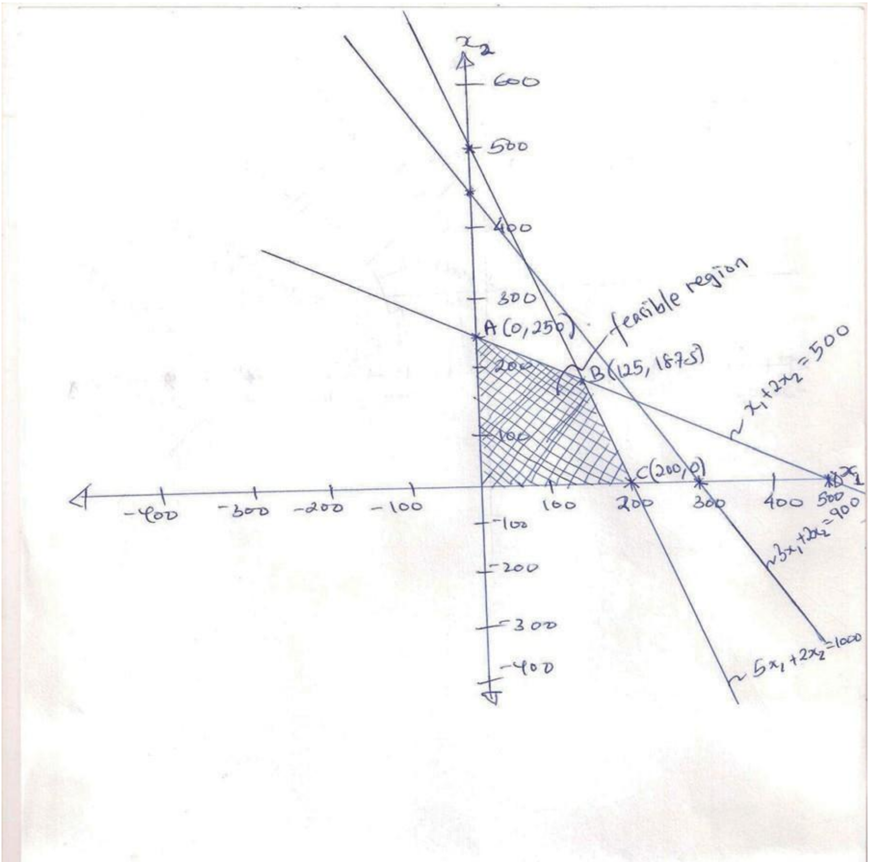
$$x_1, x_2 \geq 0$$

Solution:

- To represent the constraints graphically the inequalities are written as equalities.
- Every equation is represented by a straight line.
- To draw the lines, two points on each of the lines are found as indicated in the following table (intercepts);
-

Equation	x_2 intercept when $x_1 = 0$	x_1 intercept when $x_2 = 0$	Point (x, y) on the line
$5x_1 + 2x_2 = 1000$	$x_2 = 500$	$x_1 = 200$	(0,500)(200,0)
$3x_1 + 2x_2 = 900$	$x_2 = 450$	$x_1 = 300$	(0,450)(300,0)
$x_1 + 2x_2 = 500$	$x_2 = 250$	$x_1 = 500$	(0,250)(500,0)

and $x_1 = 0$, x_1 axis $x_2 = 0$, x_2 axis. Plot each equation on the graph.



x_2
600
500
400
300
200
100
-100
-200
-300
-400
x_1
-400
-300
-200
-100
100
200
300
400
500
A (0,250)
B (125, 187.5)
C (200, 0)
feasible region
$x_1+2x_2=500$
$3x_1+2x_2=900$
$5x_1+2x_2=1000$

B is the point of intersection of lines $x_1 + 2x_2 = 500$, $5x_1 + 2x_2 = 1000$ on solving we get $B = (125, 187.5)$

Corner Points	Value of $Z = 100x_1 + 40x_2$
$A(0, 250)$	10,000
$B(125, 187.5)$	20,000(Max. Value)
$C(200, 0)$	20,000(Max. Value)

Therefore, the Maximum value of Z occurs at two vertices B and C gives the maximum value of Z. Thus, there are multiple optimum solution for the LPP.

2. Ubounded Solutions:

Example: Use graphical method to solve the following LPP.

$$Max(Z) = 3x_1 + 2x_2$$

Subject to

$$5x_1 + x_2 \geq$$
$$10\ x_1 + x_2 \geq$$
$$6\ \ x_1 + 4x_2 \geq$$
$$12$$

and

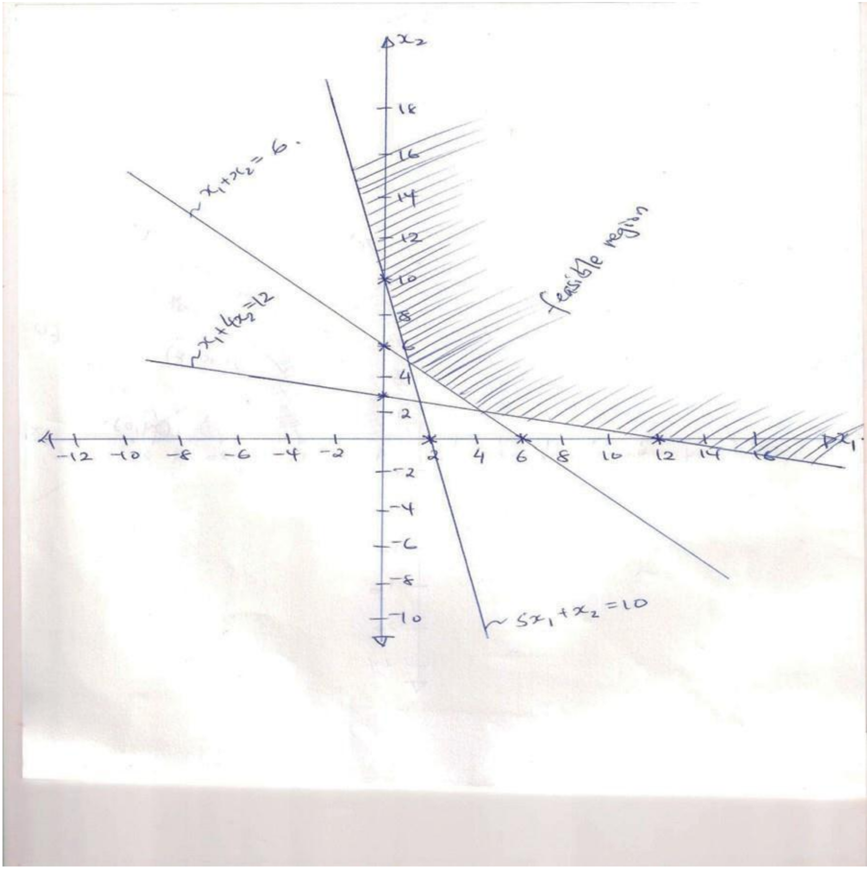
$$x_1, x_2 \geq 0$$

Solution:

- To represent the constraints graphically the inequalities are written as equalities.
- Every equation is represented by a straight line.
- To draw the lines, two points on each of the lines are found as indicated in the following table (intercepts);

Equation	x_2 intercept when $x_1 = 0$	x_1 intercept when $x_2 = 0$	Point (x, y) on the line
$5x_1 + x_2 = 10$	$x_2 = 10$	$x_1 = 2$	(0,10)(2,0)
$x_1 + x_2 = 6$	$x_2 = 6$	$x_1 = 6$	(0,6)(6,0)
$x_1 + 4x_2 = 12$	$x_2 = 3$	$x_1 = 12$	(0,3)(12,0)

and $x_1 = 0$, x_1 axis $x_2 = 0$, x_2 axis. Plot each equation on the graph.



The feasible region is unbounded. Thus, the maximum value of Z occurs at infinity; hence, the problem has an unbounded solution.

3. No Feasible Solution:

Example: Use graphical method to solve the following LPP.

$$Max\ (Z) = x_1 + x_2$$

Subject to

$$x_1 + x_2 \leq 1$$
$$-3x_1 + x_2 \leq 3$$

and

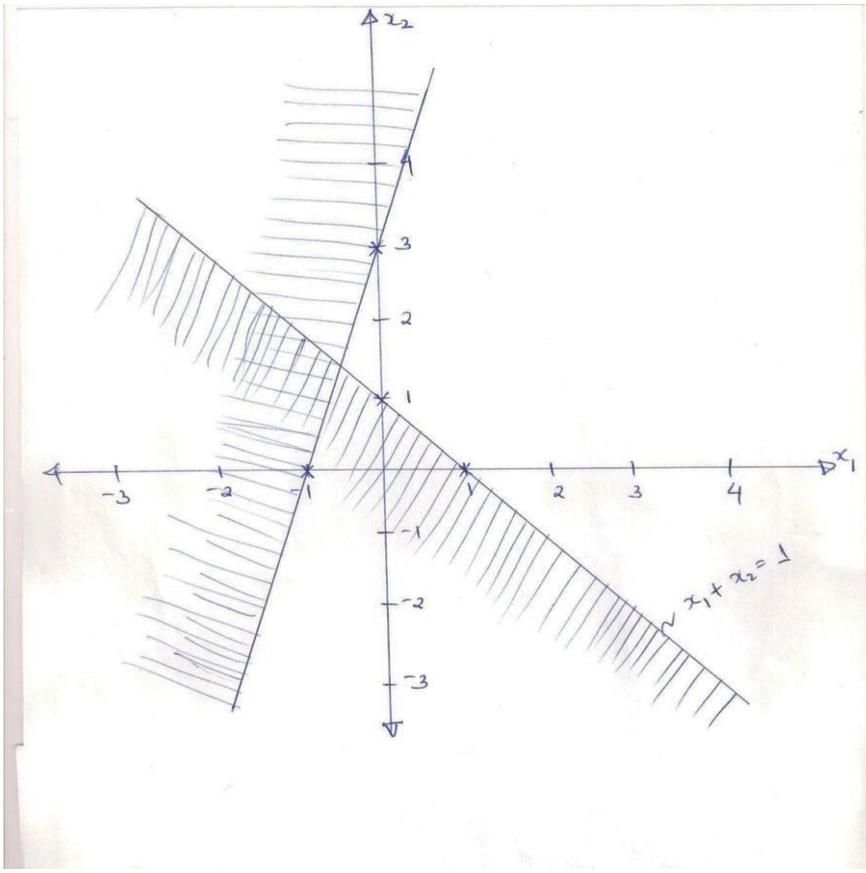
$$x_1, x_2 \geq 0$$

Solution:

- To represent the constraints graphically the inequalities are written as equalities.
- Every equation is represented by a straight line.
- To draw the lines, two points on each of the lines are found as indicated in the following table (intercepts);

Equation	x_2 intercept when $x_1 = 0$	x_1 intercept when $x_2 = 0$	Point (x, y) on the line
$x_1 + x_2 = 1$	$x_2 = 1$	$x_1 = 1$	(0,1)(1,0)
$-3x_1 + x_2 = 3$	$x_2 = 3$	$x_1 = -1$	(0,3)(-1,0)

and $x_1 = 0$, x_1 axis $x_2 = 0$, x_2 axis. Plot each equation on the graph.



In the above graph, there being no point (x_1, x_2) common to both the shaded regions. We cannot find a feasible region for this problem. So the problem can not be solved, hence, the problem has no solution.